One-Fluid Model for Plasma 2: The Conclusion

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Outline

- One-fluid model
- Generalized Ohm's law
- Hall effect
- Basic idea of MHD

One-fluid Model

- For length-scales larger than Debye length and timescales larger than the inverse of plasma frequency, charge separation can be neglected and a fully ionized plasma can be regarded as a single fluid.
- Collisions cannot be negelcted in these largescales.

Consider an electron fluid. Due to collisions, the momentum transfer is dominant between electrons and ions, than among the electrons.

- This loss of momentum for an electron due to an encounter with an ion is $m_e n(v_e - v_i)$
- Equation of motion for an electron fluid is given by,

$$m_e n \frac{\partial v_e}{\partial t} = -\nabla p_e - ne(E + \frac{v_e}{c} \times B) - m_e n \nu_c (v_e - v_i) \quad (1)$$

- Current density relation

$$j = ne(v_i - v_e) \quad (2)$$

- Substituting Eq. 1 in Eq. 2,

$$m_e n \frac{\partial v_e}{\partial t} = -\nabla p_e - ne(E + \frac{v_e}{c} \times B) + ne\eta \mathbf{j}, \qquad (3)$$

- where η is given by

$$\eta = \frac{m_e \nu_c}{ne^2} \qquad (4)$$

- the collision term in the equation of the ion fluid should be equal to that for the electron fluid in (3) with an opposite sign, i.e

$$m_i n \frac{\partial v_i}{\partial t} = -\nabla p_i + ne(E + \frac{v_i}{c} \times B) - ne\eta \mathbf{j}, \qquad (5)$$

- On adding the equations 3, 5 we get

$$n\frac{\partial}{\partial t}(m_i v_i + m_e v_e) = \frac{ne(v_i - v_e)}{c} \times B - \nabla(p_i + p_e) \qquad (6)$$

- The density ρ and the fluid velocity v in the single-fluid model are given by

$$\rho = n(m_i + m_e) \qquad (7)$$

$$v = \frac{(m_i v_i + m_e v_e)}{m_i + m_e} \qquad (8)$$

- Using the equations 6, 7 and 8 we get,

$$\rho\frac{\partial v}{\partial t} = \frac{1}{c} j \times B - \nabla P \qquad (9)$$

- To obtaine the relation we have considered,

$$p = p_i + p_e \tag{10}$$

- Now multiply Eq. 3 by m_i and subtract it from Eq. 5 multiplied by m_e. This gives,

$$\begin{aligned} m_i m_e n \frac{\partial(v_i - v_e)}{\partial t} = ne(m_i + m_e)E + \frac{ne}{c}(m_e v_i + m_i v_e) \times B \\ - m_e \nabla P_i - m_i \nabla P_e - (m_i - m_e) ne\eta j \end{aligned} \tag{11}$$

- Using the Equations (2),(7),(8) we can obtain the relation ,

$$\begin{aligned} m_e v_i + m_i v_e &= m_i v_i + m_e v_e + (m_e - m_i)(v_i - v_e) \\ &= \frac{\rho}{n} v + \frac{m_e - m_i}{ne} j \end{aligned} \tag{12}$$

- Using this with the Eq 11 we get , The generalised Ohms law

$$E + \frac{v}{c} \times B - \eta j = \frac{1}{e\rho}[\frac{m_i m_e n}{e} \frac{\partial}{\partial t}(\frac{j}{n}) + (m_e - m_i)\frac{j}{c} \times B + m_e \nabla P_i - m_i \nabla P_e \tag{13}$$

- The equation then simplifies into

$$E + \frac{v}{c} \times B - \eta j = \frac{1}{ne}[\frac{j}{c} \times B - \nabla P_e] \quad (14)$$

- Term involving $j \times B$ corresponds to the Hall effect
- The equation can be further simplified into Ohm's law

$$j = \sigma(E + \frac{v}{c} \times B) \quad (15)$$

where electrical conductivity,

$$\sigma = \eta^{-1} \quad (16)$$

- The single-fluid model of the plasma is known as MHD.
- $$j = \sigma(E + \frac{v}{c} \times B) \tag{17}$$
  $$\rho\frac{\partial v}{\partial t} = \frac{1}{c}j \times B - \nabla P \tag{18}$$
  These equations used in further development of MHD in the later chapter
- The equations of continuity for the electron and ion fluids give the one-fluid equation of continuity

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho v) = 0 \tag{19}$$