One-Fluid Model for Plasma

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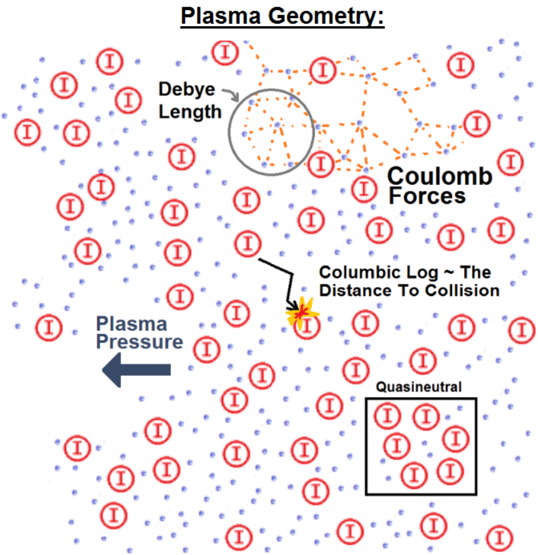
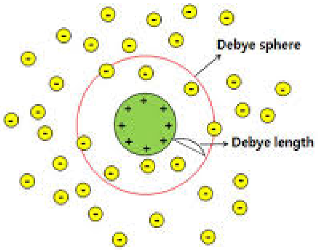
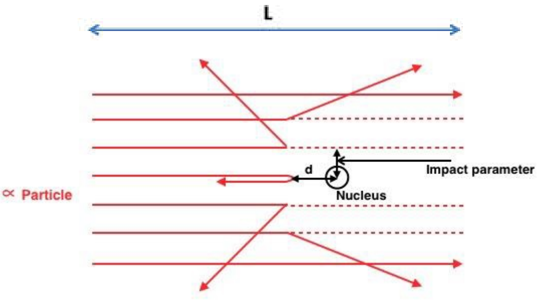
Collisions in Fully Ionised Plasma

- The force of interactions between the particles follow the inverse-square law.
- The force due to a charged particle is screening beyond a length, which is called **Debye length**.
- If the impact parameter of the collision is large, the deflection is small
- And we are interested in r$\leq r_0$, defined as the impact parameter when $\delta p \approx p$.


Plasma Geometry:
Debye Length
Coulomb Forces
Columbic Log ~ The Distance To Collision
Plasma Pressure
Quasineutral


Debye sphere
Debye length


L
d
Impact parameter
Nucleus
∝ Particle

Let's make a rough estimate of r_0 and collision frequency ν_c

- If relative velocity between the particle is u, then r_0/u is the time during which particle will be close to make strong interaction.
- The impulse is of the order of

$$\frac{e^2}{r_0^2}.\frac{r_0}{u} \approx \frac{e^2}{r_0 u} \approx \delta p$$

$$r_0 \approx \frac{e^2}{m_e u^2}$$

- Now, collision cross-section is πr_0^2. A particle moving with u velocity will collide with particles inside the cylinder of volume $\pi r_0^2 u$ in unit time.

- If n is the number density, then the collision frequency is given as,

$$\nu_c \approx \pi r_0^2 u \approx \frac{\pi n e^4}{m_e^3 u^3} \tag{1}$$

- If $u \approx (k_B T / m_e)^{1/2}$,

$$\nu_c \approx \frac{\pi n e^4}{m_e^{1/2} (k_B T)^{3/2}} \tag{2}$$

One-fluid Model

- For length-scales larger than Debye length and timescales larger than the inverse of plasma frequency, charge separation can be neglected and system is quasi-neutral.
- In these scales, a fully ionized plasma can be regarded as a single fluid.
- Collisions cannot be negelcted in these largescales.
- So first we consider how to include collisions in the two fluid model and then come to a single-fluid model.

Consider an electron fluid. Due to collisions, the momentum transfer is dominant between electrons and ions, than among the electrons.

- This loss of momentum for an electron due to an encounter with an ion is $m_e n(v_e - v_i)$
- Equation of motion for an electron fluid is given by,

$$m_e n \frac{\partial v_e}{\partial t} = -\nabla p_e - ne(E + \frac{v_e}{c} \times B) - m_e n \nu_c (v_e - v_i) \qquad (3)$$

- We have current density,

$$\mathbf{j} = ne(v_e - v_i) \tag{4}$$

- Substituting Eq. 4 in Eq. 3,

$$m_e n \frac{\partial v_e}{\partial t} = -\nabla p_e - ne(E + \frac{v_e}{c} \times B) + ne\eta\mathbf{j}, \tag{5}$$

- where η is given by

$$\eta = \frac{m_e \nu_c}{ne^2} \tag{6}$$

The physical significance of η can be understood as,

- Consider a unmagnetized plasma in steady state. The electric field **E** here only opposes the current **j**.
- Eq.5 becomes,

$$-neE + ne\eta j = 0 \tag{7}$$

$$E = \eta j \tag{8}$$

- This implies that η is the electrical resistivity of the plasma. Using Eq 2, we can write η as,

$$\eta = \frac{\pi m_e^{1/2} e^2}{(k_B T)^{3/2}} \tag{9}$$

- A rigorous analysis was done by Spitzer and Harm(1953) for resistivity as,

$$\eta = \frac{\pi^{3/2} m_e^{1/2} Z e^2}{(2\gamma_E^2 k_B T)^{3/2}} \ln \Lambda \tag{10}$$

where γ_E is a parameter depending on Z and is 0.58 for Z=1. λ is given by,

$$\Lambda = \frac{3}{2Ze^2} \left(\frac{k_B^3 T^3}{\pi n} \right)^{1/2} \tag{11}$$

.

- Substituting the values of the constants, Spitzer conductivity is given by,

$$\eta = 7.3 \times 10^{-9} \frac{ln\Lambda}{T^{3/2}} \quad e.s.u \tag{12}$$